# Joint seismic traveltime and waveform inversion for near surface imaging

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#### Summary

The first-arrival traveltime tomography is currently a standard approach for imaging the near surface structures. But it cannot resolve complex situation such as hidden lowvelocity layers. Early arrival waveform inversion is a robust approach for dealing with complex structures, but it may take significant computational efforts. Furthermore, in practice, we found that the results from waveform inversion may not allow fitting the first arrival traveltimes anymore because of nonuniqueness of the model solutions. These theoretical and practical issues motivated us to develop joint first arrival traveltime and waveform inversion. We apply a regularized nonlinear conjugate gradients method to simultaneously invert both traveltime and waveform data. One of the difficulties for performing waveform inversion alone is the lack of effective preconditioning in nonlinear inversion. Therefore, it requires hundreds of iterations to converge to a desired minimum misfit. With the inclusion of traveltime data in joint inversion, however, the inverse matrix of traveltime sensitivity could serve as an effective preconditioner to waveform inversion. Thus, it helps significantly speeding up waveform inversion. We demonstrate the effectiveness of the joint inversion by applying to both synthetics and real data.

### Introduction

The first-arrival traveltime tomography has been widely applied today for near surface imaging. Full waveform inversion (FWI) including near surface applications is emerging and presenting a great potential for looking into further details in the earth models (Tarantola, 1984; Pratt et al., 1998; Sheng et al., 2006; Zhang and Zhang 2012). Near surface problems, on the other hand, could be far more complex than what we can handle today, demanding more advanced technologies (Keho and Kelamis, 2012).

inverse problems fundamentally Geophysical are nonunique (Aki and Richards, 1980). These include traveltime tomography and full waveform inversion. That means there could be many global solutions that match data equally well. While seeking additional geophysical or geological constraints, we develop a joint traveltime and waveform inversion method for imaging the near surface area and intend to constrain the solutions by combining two types of data and two different imaging technologies (see Figure 1). The first arrival traveltimes are independent attributes out of the original data, while the early arrival waveforms are also associated with the near surface structures with profound information. But fitting waveforms may or may not honor the first arrival traveltime fit, especially when dealing with data with noise. By joint inversions, we hope to fit both data with different physical imaging theories. Meanwhile, our concern is that the computational effort for joint inversion could become much bigger, and joint inversion could take much longer time to complete. Nevertheless, with inclusion of the traveltime inversion, the waveform inversion actually converges much faster. This is because traveltime raypaths could help waveform inversion finding solutions quicker by preconditioning the gradient of the objective function.

#### Nonunique Near Surface Velocity Solutions



Figure 1: Schematic plot explaining the idea of joint inversion for constraining nonunique near surface velocity solutions.

### Joint inversion algorithm

We impose the following objective function for joint traveltime and waveform inversion:

$$\phi(\mathbf{m}) = (1 - \omega) \|\mathbf{P}_{o} - \mathbf{P}_{s}(\mathbf{m})\|^{2} + \omega \|\mathbf{t}_{o} - \mathbf{t}_{c}(\mathbf{m})\|^{2} + \tau \|\mathbf{L}(\mathbf{m} - \mathbf{m}_{o})\|^{2}$$
(1)

where  $P_0$  is waveform data,  $P_s$  is synthetic waveform,  $t_0$  is picked traveltime,  $t_c$  is calculated traveltime, m is the velocity model,  $m_0$  is *a priori* model. L is a Laplacian operator for regularization,  $\omega$  is a scaling factor between waveform misfit and traveltime misfit.

We apply a nonlinear conjugate gradient method to minimize the above objective function, and calculate the following gradient that will determine the model update direction:

$$g(\mathbf{m}) = (1 - \omega) \dot{\mathbf{P}}_{F} \dot{\mathbf{P}}_{B} - \omega \mathbf{A}^{\mathrm{T}} (\mathbf{t}_{a} - \mathbf{t}_{c}(\mathbf{m})) + \tau \mathbf{L}^{\mathrm{T}} \mathbf{L} (\mathbf{m} - \mathbf{m}_{a})$$
(2)

where  $\mathbf{P}_F$  and  $\mathbf{P}_B$  are the forward and backward propagation wavefield for imaging that provides with sensitivity impact

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and directs waveform inversion, and  $\mathbf{A}^{T}$  is a transposed sensitivity matrix of traveltimes containing raypath information, equivalent to the impact of traveltime sensitivity. For the traveltime inverse problem, however, we can easily access both  $\mathbf{A}^{T}$  and A matrix after raytracing. Thus, we are able to place the following preconditioner to the gradient iteratively:

$$\mathbf{p}_{k} = \mathbf{P} \,\mathbf{g}_{k} = (\omega A^{T} A + \tau L^{T} L + \varepsilon_{k} I)^{-1} \,\mathbf{g}_{k}$$
(3)

Where  $\mathbf{g}_k$  is the gradient for the *k-th* iteration, **P** is a preconditioner from traveltime tomography, and  $\mathbf{P}_k$  is a preconditioned gradient. Applying this traveltime preconditioner should help finding the solutions to problem (1) quicker than by using the raw gradient alone.

Next, we should determine step length for model updates and that involves forward modeling for both waveform and traveltimes and ensures the update honoring both data.

### Synthetic example

We design a numerical experiment to test joint inversion. The true model is shown in Figure 2a. It includes low velocity layers underneath a high velocity layer, a low velocity zone, and a velocity gradient zone. Figure 2b shows the result from traveltime tomography. It could not resolve the low velocity layers below the top high velocity layer. FWI could establish the low velocity zone (Figure 2c), but the velocity of the top layer seems too high. Joint traveltime and waveform inversion keeps most of the features in FWI results, but lowers the near surface velocities so that allows fitting the first arrival traveltimes (Figure 2d).



Figure 2: Synthetic experiment: a) true model; b) traveltime tomography result; c) waveform inversion result; d) joint traveltime and waveform inversion.

The advantage of joint inversion is obvious, although the final image may not reconstruct the true model perfectly. It suffers from edge effects at both sides in the model. In fact, any velocity reversal problem is a serious challenge for near surface imaging. Joint inversion improves the solution.

#### Real data example

We also apply the method to real data on a 2D line. It consists of 243 shots with shot spacing of 40 m. There are 400 channels with receiver spacing of 20 m. The surface topography varies but not significantly.



Figure 3: a) Initial velocity model; b) traveltime tomography solution; c) waveform inversion solution; d) joint traveltime and waveform inversion solution.

We build a layer starting model by picking refraction turning points from the first arrival traveltimes (Figure 3a). The first arrival traveltime tomography generates a velocity solution shown in Figure 3b. Following the first arrival

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picks on the waveform data, we keep a window of 250 ms long for early arrivals and mute the remaining data for waveform inversion. Figure 3c shows the waveform inversion result. It presents more velocity details in the model than the traveltime tomographic solution.

Figure 4a displays one of the shot gathers and compares data (black) with the corresponding synthetic waveform (red) after waveform inversion. Because the amplitude of the first arrivals is very small comparing to the later arrivals, synthetic waveform does not actually match the very first arrival. A secondary arrival seems mismatching the wrong phase as well. But the overall waveform fit is reasonable.

a) Waveform inversion result



b) Joint inversion result



Figure 4: a) synthetics (red) and input data (black) after 9 iterations from waveform inversion; b) synthetics (red) and input data (black) after 9 iterations from joint inversion.

Figure 3d shows the result from joint traveltime and waveform inversion with subtle differences from waveform inversion alone. The relative deep model further varies, but

the top near surface area shows similar velocities to the traveltime tomography results. This is particularly true in circled area. In this area, waveform inversion departs from the traveltime tomographic solution and produces higher velocity, but joint inversion clearly brings that back.

Figure 4b shows the overlay of a shot gather (black) with synthetic waveform (red) and calculated traveltimes (blue) after nine iterations of joint inversion. Not only the first arrival traveltimes are better matched, the match of amplitude and phase of later arrivals is also improved.

## Conclusions

We develop a joint first arrival traveltime and waveform inversion approach for imaging the near surface velocity structures. We test synthetic data and apply to real data. Our initial experiences with the approach are encouraging. The approach establishes a stepping stone between traveltime tomography and full waveform inversion. It gives us an opportunity to study the issues between fitting traveltimes and fitting waveforms.

Both the first arrival traveltimes and the early arrival waveforms are reliable sources of data. There should be a plenty of real data to apply the approach for imaging the near surface velocity structures. With the inclusion of raytracing results for inversion, the joint inversion problem can be preconditioned, thus it is computationally faster to converge than waveform inversion alone. Joint inversions help constraining the nonunique solutions. However, nonuniqueness is a fundamental issue in our data. Joint inversion cannot solve the problem. One must seek other geological or geophysical constraints for more accurate solutions.

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### EDITED REFERENCES

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# REFERENCES

Aki, K., and P. G. Richards, 1980, Quantitative seismology: W. H. Freeman.

- Keho, T., and P. G. Kelamis, 2012, Focus on land seismic technology: The near-surface challenge: The Leading Edge, **31**, 62–68, http://dx.doi.org/10.1190/1.3679329.
- Pratt, R. G., C. Shin, and G. Hicks, 1998, Gauss-Newton and full-Newton methods in frequency-space seismic waveform inversion: Geophysical Journal International, **133**, no. 2, 341–362, http://dx.doi.org/10.1046/j.1365-246X.1998.00498.x.
- Sheng, J., A. Leeds, M. Buddensiek, and G. T. Schuster, 2006, Early arrival waveform tomography on near-surface refraction data: Geophysics, 71, no. 4, U47–U57, <u>http://dx.doi.org/10.1190/1.2210969</u>.
- Tarantola, A., 1984, Inversion of seismic reflection data in the acoustic approximation: Geophysics, **49**, 1259–1266, <u>http://dx.doi.org/10.1190/1.1441754</u>.
- Zhang, W., and J. Zhang, 2011, Full-waveform tomography with consideration for large topography variations : 81<sup>st</sup> Annual International Meeting, SEG, Expended Abstracts, **30**, 2539–2542.